

N. Vavilov and Digital Foundations of Mathematics and Mathematical Education

Alexei Semenov

Abstract. The talk addresses activities of the outstanding mathematician Nikolai Alexandrovich Vavilov and the results of this activity, which lie outside the theorems proved by him and the definitions proposed by him.

I will talk about three areas that are connected and intersect in Vavilov's world:

- Computer in mathematical research
- Philosophy of mathematics and foundations of mathematics
- The educational meaning and significance of mathematics

In addition to personal communication with N.A., I rely primarily on his works, a list of which is given at the end of this abstract.

In particular, among them is a review of the applications of the computer for research in number theory [1, 2, 3, 4, 5, 6]. The total number of references read by Nikolai in this review is 1765.

Note that Vavilov's views on the use of computers in mathematics and mathematical education are based on technologies mainly formed in the 1980s (or early 1990s). I consider this as a positive factor in our discussion. The first technology is a mathematical text editor designed by one person — Donald Knuth; this editor has fundamentally reduced the distance between the mathematician who wrote a text and the mathematician who reads it. The second are computer algebra systems that have reduced the distance between the research mathematician and the mathematical reality under observation and study. Vavilov mentions and used various CAS, but Mathematica was still the main resource for him in the educational field. It was also designed by one man, Steven Wolfram. In a sense, considering this stage of technology is important to us today.

We have to understand clearly what can be done successfully, BEFORE the advent of big data and big players in the field. As you know the results of the advent are being published in Nature with 5–10 authors from a leading corporation and a top university.

In the talk, I will try to analyze the oppositions and relationships formulated by Vavilov based on decades of rational AI preceding the years of intuitive AI and creative AI. Among them:

- An ideal mathematical reality and an experiment in it
- Coordination of ideas and calculations, theory and experiment
- Deterministic and random
- The simplicity of a mechanism for generation and the unpredictable complexity of the generated
- Various finitnesses, including virtually infinite and intermediate ones
- Proof and error in mathematics
- etc.

In the talk, I will present my concept of the complexity of objects, which is relevant to the discussion about different finitnesses. I propose to consider the complexity of the finite object relative to a mode (program) of description as the sum of the length of the mode and the length of the shortest description. With this definition, the optimal way of describing will be worse by only additive dozens, or at least hundreds, than other, as Kolmogorov believed. So, when talking about 'not-so-real' numbers I propose to consider numbers not being big (Knuth a.o.) but having high complexity

Many fundamental positions related to the philosophy of mathematics and mathematical activity, the role of evidence, proof, and errors in this activity are reflected in [7]. In connection with the discussion of our ideas about mathematical proofs, I propose an interpretation of Hilbert's program as a goal to prove that there is no contradiction in mathematics with complexity less than the size of the Universe.

Another fundamental text by Vavilov [8] refers to his educational principles. I forgot to ask Nikolai Alexandrovich about the meaning of the initial metaphor of the title. I hope his co-authors will explain this to me at the conference. My, external reconstruction of meaning: "Don't panic!" Vavilov says that "Teaching mathematics should intrigue, captivate and fascinate". I will try to point out two features of learning aimed at implementing this principle for all students: solving problems that are not known how to solve, and the ability to use digital technologies.

The speaker hopes for the active participation of listeners, in particular colleagues and co-authors of Nikolai Vavilov in his educational and other endeavors. Such participation will make it possible to more fully restore, preserve and develop Nikolai Vavilov's legacy in the areas.

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Alexei Semenov

Dept. of Mathematical Logic and Theory of Algorithms

Lomonosov Moscow State University

Moscow, Russia

e-mail: alexei.semenov@math.msu.ru